General Comments on the Florida Common Core Mathematics Standards

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I am a visiting scholar at the Hoover Institution at Stanford university. Between 2007 and 2009 I served as a senior policy adviser at the U.S. Department of Education reviewing all the Department's evaluation studies except those produced by IES. I have been a member of the Content Review Panel for the California state test since its inception in 1998, and I served as a commissioner on the California Academic Content Standards Commission that in 2010 evaluated the Common Core's suitability for California. I have authored multiple academic studies evaluating the Common Core mathematics standards.

The Florida Common Core standards make the following major claims.

- That the standards are rigorous and internationally benchmarked;
- That the standards reflect college readiness and will reduce college remediation;
- That the standards will improve American students' academic achievement, and lead more students to succeed in STEM careers.

I will not touch here on the secretive and insular process used to write the Common Core standards with minimal public input. Instead, I will focus solely on their academic quality. In an appendix to this testimony I offer a list of comments on individual Florida Common Core standards in support of the general observations I make here.

1. The rigor of the Common Core mathematics in the context of international expectations.

To understand the many seemingly conflicting opinions about the Common Core, ranging from claims that they are exceedingly demanding and developmentally inappropriate, to arguments that the Common Core mathematics trails international high achievers by a year or two, one needs to understand that both claims can be simultaneously true. The reason is that the Common Core mathematics is highly uneven across grades and in its coverage across content areas. To see the overall nature of those standards one needs to examine their flow in detail.

The Common Core starts very aggressively in Kindergarten, often exceeding what international high achievers expect of their children. For example, Common Core expects kindergartners to characterize and classify complex two and three-dimensional shapes such as hexagons, cylinders, cubes, or cones; it expects kinders to translate concrete addition and subtraction situations to written number sentences and equations. Similarly, it expects first graders to compose and decompose two- and three-dimensional shapes into more complex shapes, or understand abstract transitivity – for example, that if A is larger than B and B is larger than C, than it must follow that A is always larger than B. Such skills are a year or

more ahead of what is broadly expected across the world and are bound to unnecessarily stress teachers and students.

This "fast and furious" pace quickly slows down in grades 2 and beyond, where students endlessly repeat arithmetic standards that expect them to apply this "property" to that particular operation in one way, then next grade to apply another "relationship" to the same operation, to yet another "strategy" the following grade to explore the same. While this endless meandering takes place in grades 2 through 6 with arithmetic of integers and fractions, the Common Core neglects to develop supporting algebraic concepts such as the fact that when equals are added to equals the results stay equal, understanding of the area of a triangles, the circumference of a circle, or the sum of angles in a triangle. Even a key fraction skill – converting between common fractions, decimals and percent – barely shows up in the standards, despite fractions being supposedly the focus of the Common Core. As a consequence, by the middle school the Common Core finds itself a year or two behind what high achieving countries teach their children. Worse, even when the Common Core belatedly expects American students to master arithmetic with the standard algorithms, they are bound to be confused because of the plethora of indiscriminate "strategies" and "models" they spent their time on in prior years. Small wonder that Andrew Porter, the Dean of University of Pennsylvania Graduate School of Education, noted with surprise in his evaluation of the Common Core that "**High-performing countries' emphasis on 'perform procedures' runs counter to the widespread call [in the Common Core] for a greater emphasis on higher order cognitive demand**."

Three other comparative studies found essentially the same: that despite Common Core's aggressive start in kindergarten, by grade 8 it falls by a year or more behind international high achievers. Nowhere is it more clearly visible than by Common Core's reversal of our own long-standing efforts of having more 8th graders study algebra: Common Core reversed this trend and firmly pushed Algebra 1 back to the high school.

Only a single recent study of Bill Schmidt and Richard Houang from Michigan State University claims to have found the opposite: that the Common Core is on par with other high achieving countries. Yet if one carefully reads their paper it becomes painfully obvious that even their own data doesn't support their paper's conclusions, which are based mostly on sleight-of-hand visual manipulation.

In fact, it turns out the main value of the Schmidt & Houang study is to illustrate how incoherent and fragmented Common Core mathematics actually is when compared to the content sequence of high achieving countries

2. Common Core's college-readiness.

A major claim of the Common Core is that it will prepare all students to be college-ready. Yet Jason Zimba, a key author of the math standards, clearly acknowledged that this readiness is at most for non-selective community colleges. In his words: "**for the colleges most kids go to, but not for the colleges most parents aspire to**."

This is not really surprising to anyone who had looked closely at Common Core's high school mathematics. Common Core's definition of college-readiness claims to include content of Algebra 2 and Geometry, yet its definition of those courses includes only a fraction of their traditional content. Arithmetic and Geometric series are barely touched on, logarithms and logarithmic functions are undeveloped, as are conic sections, polar coordinates and functions, or mathematical proofs.

Full Algebra 2 content is barely sufficient for a four-year college entry to being with, as is clearly demonstrated by California Early Access Program that has been testing student readiness for the California State University (CSU) system for almost a decade. Less than 30% are fully or partially ready for CSU after Algebra 2, in contrast to about 90% being ready after an additional math course.

Again, this shouldn't surprise anyone. Common Core replaced high school diploma with its notional "college readiness" certificate to graduate high school. Yet unless one believes that close to 100% of seniors can become truly college ready, something had to give: it is politically unthinkable that we will fail half or more of our high school seniors because they won't be deemed college ready. Consequently, Common Core had no choice but to offer a fake definition of college-readiness.

In other words, the hope that Common Core's college-readiness will reduce college remediation seems misplaced. If at all, it is bound to increase remediation rates when large number of seniors declared college-ready by fiat will head for four-year colleges.

3. Improvement in STEM preparation and STEM success.

It is sadly ironic, but STEM readiness is bound to be the biggest casualty of the Common Core.

To be prepared for a STEM career, students need to enter college at least ready to take calculus. For more selective colleges and more demanding STEM careers, freshmen must start with advanced calculus in college. This is undisputable, and widely known. Jason Zimba again: "**[Common Core is] not only not for STEM, it's also not for selective colleges. For example, for UC Berkeley, whether you are going to be an engineer or not, you'd better have precalculus to get into UC Berkeley."** Just to make it crystal clear, he recently repeated it: "**If you want to take calculus your freshman year in college, you will need to take more mathematics than is in the Common Core**."

Indeed, to get to calculus in their senior year, students need to start with Algebra 1 in the 8th grade. Even to get to pre-calculus the Common Core is missing large chunks of necessary content as I described above. It is, therefore, unsurprising, that the College Board Vice President in charge of AP Calculus recently declared that "**AP Calculus is in conflict with the Common Core** … **and it lies outside the sequence of the Common Core because of the fear that it may unnecessarily rush students into advanced math classes.**"

Does it mean that nobody will take calculus anymore in high school, or be ready for STEM careers? Clearly not. Children of affluent parents will keep on getting extra-curricular support and tutoring for acceleration, as their parents are keenly aware of what is really needed for such careers. Such parents will also insist that school in their affluent neighborhoods will keep providing advanced content and

offer acceleration. The hardest hit group will be students from challenged backgrounds, whose families tend to be unaware of what is actually needed for STEM careers, whose parents have difficulty providing the needed extra-curricular support, and who attend schools that cannot afford to offer much beyond the Common Core.

In other words, the Common Core actually stacks the deck against disadvantaged students. They will no longer be able to rely on schools to provide the needed content as a part of school's regular curriculum; all the while they will be told that they are "on track to be college ready."

A cruel joke indeed.

If Florida wants to prepare more students for academic careers, if Florida wants to broaden the access to STEM for more students from disadvantaged background, if Florida wants to reduce the achievement gap by raising the floor rather than by lowering the ceiling, Florida would be well advised to take excellent and proven state standards such as Massachusetts' or Indiana's, and at most tweak them slightly to adjust for Florida's unique circumstances.

Thank you for your time.

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Appendix

Kindergarten

MACC.K.CC.1.1
Strand: Counting and Cardinality
Standard: Know number names and the count sequence
Description: Count to 100 by ones and by tens.
Comments:*The counting to 100 is unwisely aggressive. As a consequence, in grade 1 it is only extended to 120. A more reasonable sequence would be to count to 20 in Kindergarten and to 100 in grade 1.*

MACC.K.CC.1.2
Strand: Counting and Cardinality
Standard: Know number names and the count sequence
Description: Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
Comments:*Unwisely aggressive for numbers up to 100. A limit of 20 would be more appropriate.*

MACC.K.CC.3.7
Strand: Counting and Cardinality
Standard: Compare numbers
Description: Compare two numbers between 1 and 10 presented as written numerals.
Comments:*Bordering on inappropriate.*

MACC.K.G.1.1
Strand: Geometry
Standard: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)
Description: Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind, and next to.*
Comments:*Inappropriate for 3D shapes, and unnecessary for 2D shapes beyond rectangles (or even just squares).*

MACC.K.G.1.2
Strand: Geometry
Standard: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)
Description: Correctly name shapes regardless of their orientations or overall size.
Comments:*Inappropriate for 3D shapes, and unnecessary for 2D shapes beyond rectangles (or even just squares).*

MACC.K.G.1.3
Strand: Geometry
Standard: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)

Description: Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
Comments:*Inappropriate. Children at this age can intuit the difference between 2D and 3D but many have difficult time to verbalize it and/or visualize it.*

MACC.K.G.2.4
Strand: Geometry
Standard: Analyze, compare, create, and compose shapes
Description: Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
Comments:*Inappropriate and unnecessarily demanding.*

MACC.K.G.2.6
Strand: Geometry
Standard: Analyze, compare, create, and compose shapes
Description: Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*
Comments:*Inappropriate and unnecessarily demanding. Grade 2 standard in Singapore.*

MACC.K.MD.1.2
Strand: Measurement and Data
Standard: Describe and compare measurable attributes
Description: Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*
Comments:*Inappropriate and unnecessarily demanding. Grade 2 standard in Singapore.*

MACC.K.OA.1.3
Strand: Operations and Algebraic Thinking
Standard: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
Description: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
Comments:*Recording by equations and number sentences is inappropriate.*

MACC.K.OA.1.4
Strand: Operations and Algebraic Thinking
Standard: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
Description: For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
Comments:*Borders on educationally inappropriate, particularly the recording by equation or number sentence.*

First Grade

MACC.1.G.1.2
Strand: Geometry
Standard: Reason with shapes and their attributes
Description: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.
Comments:*Premature and inappropriate. Grade 2 standard in Singapore.*

MACC.1.G.1.3
Strand: Geometry
Standard: Reason with shapes and their attributes
Description: Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.
Comments:*Probably premature, certainly marginal. Unnecessary aggressive.*

MACC.1.MD.1.1
Strand: Measurement and Data
Standard: Measure lengths indirectly and by iterating length units
Description: Order three objects by length; compare the lengths of two objects indirectly by using a third object.
Comments:*Transitive (indirect) comparison is premature and inappropriate.*

MACC.1.MD.2.3
Strand: Measurement and Data
Standard: Tell and write time
Description: Tell and write time in hours and half-hours using analog and digital clocks.
Comments:*Missing reference to am/pm.*

MACC.1.NBT.1.1
Strand: Number and Operations in Base Ten
Standard: Extend the counting sequence
Description: Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
Comments:*Should be to 100. The wrong-headed requirement of 100 in Kindergarten forced this senseless "120" value here.*

MACC.1.NBT.2.2
Strand: Number and Operations in Base Ten
Standard: Understand place value
Description: Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones—called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

Comments:*Substandard (b) should be extended to 99 rather than 19 to support the next (1.NBT.2.3) standard.*

MACC.1.NBT.2.3
Strand: Number and Operations in Base Ten
Standard: Understand place value
Description: Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
Comments:*Cannot be done without extending 1.NBT.2.2(b) to 99.*

MACC.1.NBT.3.4
Strand: Number and Operations in Base Ten
Standard: Use place value understanding and properties of operations to add and subtract
Description: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
Comments:*Ill-defined standard open to multiple interpretations that sends mixed signals. Unclear what those "strategies based on place value, properties of operations, and/or the relationship between addition and subtraction" are. Further, if a "written method" (whatever it is) is known, why the need for those strategies? Finally, no reason to limit to a two-digit number and one-digit number if one truly expects understanding that in "adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten."*

MACC.1.OA.3.6
Strand: Operations and Algebraic Thinking
Standard: Add and subtract within 20
Description: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
Comments:*Insists on pedagogy that is appropriate for mental math but inappropriate, awkward, and constraining for written math. This standard belongs to Kindergarten. Instead, the standards should call on committing addition facts up to 20 to memory in this grade, like Singapore does in grade 1.*

Second Grade

MACC.2.G.1.1
Strand: Geometry
Standard: Reason with shapes and their attributes
Description: Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
Comments:*Badly written standard. Unclear if it refers to 2D or 3D shapes. "Faces" implies 3D, yet identification is limited to 2D shapes. Inappropriate if 3D shapes are beyond squares and rectangles.*

MACC.2.G.1.2
Strand: Geometry
Standard: Reason with shapes and their attributes
Description: Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
Comments:

MACC.2.G.1.3
Strand: Geometry
Standard: Reason with shapes and their attributes
Description: Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
Comments:*Essentially repeats the premature and inappropriate grade 1 standard MACC.1.G.1.3. OK here.*

MACC.2.MD.1.1
Strand: Measurement and Data
Standard: Measure and estimate lengths in standard units
Description: Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
Comments:*Missing "to the nearest inch/centimeter" at the end of the standard.*

MACC.2.MD.3.8
Strand: Measurement and Data
Standard: Work with time and money
Description: Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*
Comments:*Grade 2 is too late to start with money and build on children natural curiosity and their familiarity with money. It is a grade 1 standard in Singapore. In grade 2 it should include "combinations ofdollar bills, quarters, dimes, nickels, and pennies" like Singapore grade 2 does.*

MACC.2.NBT.2.7
Strand: Number and Operations in Base Ten
Standard: Use place value understanding and properties of operations to add and subtract
Description: Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
Comments:*Ill-defined standard open to multiple interpretations that sends mixed signals. Unclear what those "strategies based on place value, properties of operations, and/or the relationship between addition and subtraction" are. Further, if a "written method" (whatever it is) is known, why the need to keep relating to those strategies?If at all, standard 2.NBT.2.9 already expects this skill. Finally, it should include"fluently" as in "Fluently add and subtract within 1000," a standard in Singapore grade 2.*

MACC.2.OA.2.2
Strand: Operations and Algebraic Thinking
Standard: Add and subtract within 20
Description: Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
Comments:*One year too late. Should have been a first grade standard like in Singapore.*

MACC.2.OA.3.4
Strand: Operations and Algebraic Thinking
Standard: Work with equal groups of objects to gain foundations for multiplication
Description: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Comments:*Too low expectation. Singapore has "building up the multiplication tables of 2, 3, 4, 5 and 10 and committing to memory" as should we.*

Third Grade

MACC.3.G.1.2
Strand: Geometry
Standard: Reason with shapes and their attributes
Description: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*
Comments:*Third grade in a row repeat of grade 1 standard MACC.1.G.1.3 and grade 2 standard 2.G.1.3. Belongs to grade 2 and is below grade level here.*

MACC.3.MD.1.2
Strand: Measurement and Data
Standard: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects
Description: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
Comments:*Needs to add English units, at least oz. and lb., to the list of units to estimate.*

MACC.3.NBT.1.2
Strand: Number and Operations in Base Ten
Standard: Use place value understanding and properties of operations to perform multi-digit arithmetic
Description: Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
Comments:*At this grade high achieving countries disconnect their students from still meandering through "strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction" and simply expect their students to add and subtract within 10,000 using standard algorithms.*

MACC.3.NF.1.3
Strand: Number and Operations - Fractions
Standard: Develop understanding of fractions as numbers
Description: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

a. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
b. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

c. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Comments:*Generally OK, but the interjection of "visual fraction model" serves to confuse and should be eliminated. Number line should be used throughout.*

MACC.3.OA.1.1
Strand: Operations and Algebraic Thinking
Standard: Represent and solve problems involving multiplication and division
Description: Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
Comments:*May lead to incorrect perception with both teachers and students that the first number is always the number of groups and the second the size of the group. Needs to be adjusted to say "interpret 5 × 7 as the total number of objects in 5 groups of 7 objects eachor as 7 groups of 5 objects each"*

MACC.3.OA.3.7
Strand: Operations and Algebraic Thinking
Standard: Multiply and divide within 100
Description: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
Comments:*At this point Singapore expects "division with remainder" and "multiplication and division of numbers up to 3 digits by 1 digit" while the common core still meanders through its "strategies", "relationships," and "properties."*

Fourth Grade

MACC.4.G.1.1
Strand: Geometry
Standard: Draw and identify lines and angles, and classify shapes by properties of their lines and angles
Description: Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
Comments:*Third grade standard in Singapore.*

MACC.4.MD.3.5
Strand: Measurement and Data
Standard: Geometric measurement: understand concepts of angle and measure angles
Description: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

Comments:*The definition in (d) above implies limiting angles to integral number of degrees. Yet students already know in grade 3 that fractions exists.*

MACC.4.NBT.2.4
Strand: Number and Operations in Base Ten
Standard: Use place value understanding and properties of operations to perform multi-digit arithmetic
Description: Fluently add and subtract multi-digit whole numbers using the standard algorithm.
Comments:*Too late by a year compared to Singapore.*

MACC.4.NBT.2.5
Strand: Number and Operations in Base Ten
Standard: Use place value understanding and properties of operations to perform multi-digit arithmetic
Description: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Comments:*How would one illustrate 1275x7 using area models or arrays? Seems ill-advised at this grade level and size of numbers.*

MACC.4.NBT.2.6
Strand: Number and Operations in Base Ten
Standard: Use place value understanding and properties of operations to perform multi-digit arithmetic

Description: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Comments:*Yet again Common Core meanders around mind-numbing "strategies," "relationships," and "properties," while Singapore simply expects: "division of a 4-digit number by a 1-digit number."*

MACC.4.NF.3.6
Strand: Number and Operations - Fractions
Standard: Understand decimal notation for fractions, and compare decimal fractions
Description: Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
Comments:*Conversion between regular fractions and decimals is forgotten*.

MACC.4.NF.3.7
Strand: Number and Operations - Fractions
Standard: Understand decimal notation for fractions, and compare decimal fractions
Description: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.
Comments:*No fluency with operations with decimals is developed or expected, in contrast with Singapore, which expects at this point students to add/subtract decimals (to 2 decimal places) and multiply and divide decimals by integers. Perhaps because no concepts of money are present in the Common Core in prior years to develop foundational fluency with decimal representation.*

MACC.4.OA.2.4
Strand: Operations and Algebraic Thinking
Standard: Gain familiarity with factors and multiples
Description: Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
Comments:*Missing development of prime factorization, its uniqueness, and its uses. Consequently, Common Core never develops systematic understanding for finding common denominators or factorization.*

Fifth Grade

MACC.5.G.2.4
Strand: Geometry
Standard: Classify two-dimensional figures into categories based on their properties
Description: Classify two-dimensional figures in a hierarchy based on properties.
Comments:*Ill-defined standard that needs clarification as to what specific classes of shapes it applies to. For example, what is the hierarchy of a mix of regular and irregular polygons, or of a mix of convex and non-convex polygons?*

MACC.5.NBT.1.1
Strand: Number and Operations in Base Ten
Standard: Understand the place value system
Description: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
Comments:*Essentially a repeat of grade 4standard MACC.4.NBT.1.1*

MACC.5.NBT.2.5
Strand: Number and Operations in Base Ten
Standard: Perform operations with multi-digit whole numbers and with decimals to hundredths
Description: Fluently multiply multi-digit whole numbers using the standard algorithm.
Comments:*Grade 4 standard in Singapore.*

MACC.5.NBT.2.6
Strand: Number and Operations in Base Ten
Standard: Perform operations with multi-digit whole numbers and with decimals to hundredths
Description: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Comments:*Here again we have "strategies," "properties" and "relationships" rather than simply expecting fluency with division of integers at this grade, as Singapore does.*

MACC.5.NBT.2.7
Strand: Number and Operations in Base Ten
Standard: Perform operations with multi-digit whole numbers and with decimals to hundredths
Description: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Comments:*Here again we have "models," "drawings," "strategies," "properties," and "relationships" rather than simply expecting fluency with four operations with decimals (to three decimal places) at this grade, as Singapore does.*

Sixth Grade

MACC.6.G.1.1
Strand: Geometry
Standard: Solve real-world and mathematical problems involving area, surface area, and volume
Description: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
Comments:*Common Core has zero prior development of area of triangles yet expects them to materialize at this grade. Compare and contrast with the pedantic and tedious development of place-value or of operations with numbers. Grade 5 in Singapore.*

MACC.6.NS.1.1
Strand: The Number System
Standard: Apply and extend previous understandings of multiplication and division to divide fractions by fractions
Description: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/ bc .) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
Comments:*Unnecessary and problematic use of "visual fraction models" for division of fractions.*

MACC.6.NS.2.2
Strand: The Number System
Standard: Compute fluently with multi-digit numbers and find common factors and multiples
Description: Fluently divide multi-digit numbers using the standard algorithm.
Comments:*Grade 5 skill in Singapore.*

MACC.6.NS.2.3
Strand: The Number System
Standard: Compute fluently with multi-digit numbers and find common factors and multiples
Description: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
Comments:*Grade 5 skill in Singapore*

MACC.6.NS.2.4
Strand: The Number System
Standard: Compute fluently with multi-digit numbers and find common factors and multiples

Description: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
Comments: ***Common Core doesn't expect knowledge of prime factorization, hence it cannot expect this skill to be well supported.***

MACC.6.RP.1.3
Strand: Ratios & Proportional Relationships
Standard: Understand ratio concepts and use ratio reasoning to solve problems
Description: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Comments:***Percent is introduced in Grade 5 in Singapore.***

Seventh Grade

MACC.7.EE.2.3
Strand: Expressions & Equations
Standard: Solve real-life and mathematical problems using numerical and algebraic expressions and equations
Description: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
Comments:*Conversion between fractional forms is a key skill that is recommended to be developed around grade five by the National Mathematics Advisory Panel, by the NRC, and by NCTM. It is absent in the Common Core except for this first-time passing reference in grade 7.Also mentioned in passing in MACC.7.NS.1.2.*

MACC.7.G.1.2
Strand: Geometry
Standard: Draw, construct, and describe geometrical figures and describe the relationships between them
Description: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
Comments:*High achieving countries use constructions with compass and straightedge at this grade.*

MACC.7.G.2.4
Strand: Geometry
Standard: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
Description: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
Comments:*No prior development of the concept of Pi is present in the Common Core, here suddenly expecting students already to "know the formulas". Contrast and compare with the overly careful development of fractions in the Common Core. This skill is topic is typically addressed in grades 5 or 6 in high achieving countries.*

MACC.7.G.2.5
Strand: Geometry

Standard: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
Description: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
Comments:*Skill typically addressed in high achieving countries in grades 5 and 6.*

MACC.7.NS.1.2
Strand: The Number System
Standard: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers
Description: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.
c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Comments:*The "convert a rational number to decimal" here is one of the two places (MACC.8.NS.1.1 is the other) that conversion between common and decimalfractions iseven mentioned in passing. All other fraction conversion (decimal to common, percent to decimal or common and vice versa) are never explicitly mentioned in the Common Core*.

Eighth Grade

MACC.8.EE.1.3
Strand: Expressions & Equations
Standard: Work with radicals and integer exponents
Description: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.*
Comments:*For whatever reason Common Core refuses to call this by its standard name of Scientific Notation.*

MACC.8.G.1.2
Strand: Geometry
Standard: Understand congruence and similarity using physical models, transparencies, or geometry software
Description: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
Comments:*Experimental approach to proving congruence that failed in the few places it has been tried.*

MACC.8.G.1.4
Strand: Geometry
Standard: Understand congruence and similarity using physical models, transparencies, or geometry software
Description: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.
Comments:*Experimental approach to proving congruence that failed in the few places it has been tried.*

MACC.8.G.1.5
Strand: Geometry
Standard: Understand congruence and similarity using physical models, transparencies, or geometry software
Description: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
Comments:*The first time sum of angles in a triangle shows up in Common Core. Typically addressed in high achieving countries in grade 5 or 6.*

MACC.8.G.2.6
Strand: Geometry
Standard: Understand and apply the Pythagorean Theorem
Description: Explain a proof of the Pythagorean Theorem and its converse.
Comments:*It is unfortunate that this standard calls for "explain[ing] the proof" rather than simply for "proving" the Pythagorean Theorem.*

MACC.8.NS.1.1
Strand: The Number System
Standard: Know that there are numbers that are not rational, and approximate them by rational numbers
Description: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
Comments:*The "convert a decimal expansion which repeats eventually into a rational number" here is one of the two places (MACC.7.NS.1.2 is the other) that conversionbetweencommon and decimal fractions iseven mentioned in passing. All other fraction conversion (percent to decimal or common, and vice versa) are never explicitly mentioned in the Common Core***.**

High School Mathematics

General Comments for High School Mathematics:

- ***Missing parametric equations and functions***
- ***Missing mathematical induction***
- ***Poor coverage of complex numbers and functions***
- ***Poor coverage of polar coordinates and curves***
- ***Poor coverage of trigonometric functions***
- ***Limited content for statistics***
- ***Limited content for linear algebra***

In summary, the full content of Common Core high school mathematics is insufficient to provide the equivalent of even a strong trigonometry and linear algebra course, let alone pre-calculus.

High School Algebra

MACC.912.A-APR.1.1
Strand: Algebra: Arithmetic with Polynomials & Rational Expressions
Standard: Perform arithmetic operations on polynomials
Description: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Comments:*Division of polynomials is missing.*

MACC.912.A-APR.3.5
Strand: Algebra: Arithmetic with Polynomials & Rational Expressions
Standard: Use polynomial identities to solve problems
Description: Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
Comments:*In the original Common Core this standard has a footnote suggesting the possible use of mathematical induction. Other than that the Common Core completely forgot mathematical induction or proofs by contradiction.*

MACC.912.A-CED.1.1
Strand: Algebra: Creating Equations
Standard: Create equations that describe numbers or relationships
Description: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
Comments:*Inequalities with absolute values are missing.*

MACC.912.A-CED.1.3
Strand: Algebra: Creating Equations
Standard: Create equations that describe numbers or relationships
Description: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
Comments:*Inequalities with absolute values are missing.*

MACC.912.A-REI.2.3
Strand: Algebra: Reasoning with Equations & Inequalities
Standard: Solve equations and inequalities in one variable
Description: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
Comments:*Inequalities with absolute values are missing.*

MACC.912.A-REI.4.12
Strand: Algebra: Reasoning with Equations & Inequalities
Standard: Represent and solve equations and inequalities graphically
Description: Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Comments:*Inequalities with absolute values are missing.*

MACC.912.A-SSE.1.2
Strand: Algebra: Seeing Structure in Expressions
Standard: Interpret the structure of expressions
Description: Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.
Comments:*Misses sum and difference of cubes.*

MACC.912.A-SSE.2.3
Strand: Algebra: Seeing Structure in Expressions
Standard: Write expressions in equivalent forms to solve problems
Description: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Comments:*This standard is so generic as to be meaningless. The three specific cases are good but they don't come close to defining the specific important equivalent forms of high school mathematics. Some of the absent specific include:*

- ***sum of terms with a common factor as a multiple of a sum of terms with no common factor, e.g.,*** $xy^2 + x^2y$ ***as*** $xy\,(y + x)$
- ***product of a sum of terms as a sum of products, e.g.,*** $(x + 5)(3 - x + c)$ ***as*** $-x^2 + cx - 2x + 5c + 15$
- ***basic factoring techniques to second- and simple third-degree polynomials including finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.***

MACC.912.A-SSE.2.4
Strand: Algebra: Seeing Structure in Expressions
Standard: Write expressions in equivalent forms to solve problems
Description: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*
Comments:*Missing treatment of sums of converging infinite geometric series.Missing treatment of arithmetic series.*

High School Functions

MACC.912.F-BF.2.3
Strand: Functions: Building Functions
Standard: Build new functions from existing functions
Description: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
Comments:*This generic standard does not discriminate between applying it to quadratic functions in Algebra 1 versus general functions in more advanced courses.*

MACC.912.F-BF.2.5
Strand: Functions: Building Functions
Standard: Build new functions from existing functions
Description: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
Comments:*Common Core does not explicitly expect students to prove logarithmic relationships, just understand them.Common Core neglects teaching converting logarithms to different bases.*

MACC.912.F-IF.3.7
Strand: Functions: Interpreting Functions
Standard: Analyze functions using different representations
Description: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Comments:*Characteristics of trigonometric functions is missing phase.*

MACC.912.F-IF.3.9
Strand: Functions: Interpreting Functions
Standard: Analyze functions using different representations
Description: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Comments:*Common Core provides misses conversion of polar coordinates and curves between Cartesian and polar forms.*

MACC.912.F-LE.2.5
Strand: Functions: Linear, Quadratic, & Exponential Models
Standard: Interpret expressions for functions in terms of the situation they model
Description: Interpret the parameters in a linear or exponential function in terms of a context.
Comments:*Needs applications of quadratic equations to physics problems.*

MACC.912.F-TF.1.1
Strand: Functions: Trigonometric Functions
Standard: Extend the domain of trigonometric functions using the unit circle
Description: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
Comments:*Conversion between degrees and radians is missing.*

MACC.912.F-TF.2.5
Strand: Functions: Trigonometric Functions
Standard: Model periodic phenomena with trigonometric functions
Description: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
Comments:*Phase of trigonometric functions is missing.*

MACC.912.F-TF.3.9
Strand: Functions: Trigonometric Functions
Standard: Prove and apply trigonometric identities
Description: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Comments:*Poor coverage of trigonometric functions. Missing trigonometric functions of double angles and half angles.*

High School Geometry

MACC.912.G-CO.3.10
Strand: Geometry: Congruence
Standard: Prove geometric theorems
Description: Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
Comments:*Missing triangle inequality theorem.*

MACC.912.G-GMD.1.2
Strand: Geometry: Geometric Measurement & Dimension
Standard: Explain volume formulas and use them to solve problems
Description: Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
Comments:*Common Core neglects to treat impact of a scale factor on area, volume, or surface area.*

MACC.912.G-GPE.1.3
Strand: Geometry: Expressing Geometric Properties with Equations
Standard: Translate between the geometric description and the equation for a conic section
Description: Derive the equations of ellipses and hyperbolas given the foci and directrices.
Comments:*Common Core has minimal treatment of conic sections.It neglects completing the square to determine the nature of the equation, or to analyze the coefficient to determine characteristics such as foci, eccentricity or asymptotes.*

MACC.912.G-GPE.2.4
Strand: Geometry: Expressing Geometric Properties with Equations
Standard: Use coordinates to prove simple geometric theorems algebraically
Description: Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*
Comments:*Common core ignores proofs by contradiction.*

High School Numbers and Quantity

MACC.912.N-CN.2.4
Strand: Number & Quantity: The Complex Number System
Standard: Represent complex numbers and their operations on the complex plane
Description: Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
Comments:*Very poor coverage of polar forms. Missing conversion between functions and vectors in Cartesian and polar forms.*
